A Quantum Amplifier Circuit

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Quantum Amplifiers are an important part of superconducting quantum devices. In this paper we design a novel superconducting amplification circuit. This circuit is designed using only qubits and resonators and can therefore be intuitively understood through tracking excitations through these circuit elements. We simulate the designed circuit using QuTiP and show that the first round of amplification is 80% efficient. However, multiple rounds of amplification do not increase the expected number of photons in the signal resonator as the likelihood of relaxation surpasses amplification. This relaxation is due to the cooling qubit, which allows for excitations to leak from the storage, signal, and drive qubits. Increasing the efficiency of amplification from 80% will allow for a greater number of maximum photons in the amplification resonator before relaxation dominates behavior of the circuit.

1 Introduction

The amplification of quantum signals is important for readout in superconducting quantum processors. The current state of the art amplifier is known as the Josephson Travelling Wave Parametric Amplifier (JTWPA) [1]. This design uses four-wave mixing along two non-linearly coupled wires to amplify signal photons. Much progress has been made in this direction, with the field evolving from Josephson Parametric Amplifiers (JPA) which had a small footprint but were limited in their amplification, to JTWPA [2], and then to resonance-matched JTWPA which is the current state of the art [3].

However, understanding JTWPA requires understanding the complex Hamiltonian brought about by the capacitors, inductors, and Josephson junctions present, creating a possible barrier to entry for beginners interested in engineering superconducting circuits. In this paper we will instead construct an amplifier circuit using only qubits and resonators [4–7]. This allows a more intuitive approach to designing and understanding quantum circuits as excitations can be traced throughout the circuit's operation, rather than requiring one to engineer a more complex Hamiltonian.

Now we will begin describing the intuition behind the designed amplifier. The desired effect from amplification is to turn excitations from the drive resonator into excitations

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in the signal resonator, conditioned on there being signal present at the start of the process. Naively, this amounts to the following interaction,

$$\mathcal{H}_{int} = a^{\dagger}_{signal} a^{\dagger}_{signal} a_{drive} a_{signal} \tag{1}$$

At a first glance, this interaction allows for a drive excitation to be converted to a signal excitation, conditioned on there being a signal excitation.

However, there are a few problems with this interaction. First, it is not immediately obvious how this interaction could be engineered in a superconducting circuit, as energy conservation would require that the signal frequency ω_{signal} would have to be equal to the drive frequency ω_{drive}. Second, since interactions are reversible, this implies that the following interaction is also allowed,

$$H^{\dagger}_{int} = a^{\dagger}_{drive} a^{\dagger}_{signal} a_{signal} a_{signal} \tag{2}$$

This interaction is able to take two signal excitations and turn one of them into a drive excitation. Therefore, amplification through these interactions may not be very efficient, as the signal could leak out into the drive resonator. Even though this naive interaction has problems, we can use this as inspiration for designing a better interaction, which is not reversible and from which a superconducting circuit can be easily designed.

2 The Amplification Process

With the previous interaction as inspiration, we will solve the resonator frequency and reversibility problems by splitting the starting interaction into two separate interactions. In the process of splitting the previous interaction, we will introduce multiple new elements.

First of these new elements is the storage resonator which functions to remember if there was a signal present. In other words, the job of the storage resonator is to make the interaction conditional on there being signal present at the start. We will also introduce a cooling resonator, which functions to make the process irreversible through removing excitations from the circuit after the first step of the amplification process. Now, instead of the single interaction $a^{\dagger}_{signal} a^{\dagger}_{signal} a_{drive} a_{signal}$, we will have the three following interactions,

$$\mathcal{H}_{int1} \approx a^{\dagger}_{storage} a^{\dagger}_{cooling} a_{drive} a_{signal} \tag{3}$$

$$\mathcal{H}_{int2} \approx a_{cooling} \tag{4}$$

$$\mathcal{H}_{int3} \approx a^{\dagger}_{signal} a^{\dagger}_{signal} a_{drive} a_{storage} \tag{5}$$

The first interaction is conditioned on the signal being present, and excites the storage resonator. This interaction additionally excites the cooling resonator, which is subsequently cooled by the second interaction, making the process irreversible.

The third interaction then converts the excitation in the storage resonator into two signal excitations. Combining these interactions gives the original interaction $a^{\dagger}_{signal}a^{\dagger}_{signal}a_{drive}a_{signal}$ in an irreversible form and ensures that the signal and drive frequencies do not need to be equal, making these interactions straightforward to implement in a superconducting circuit.

To reduce the number of connections between circuit elements, we will introduce a coupling resonator, which couples all qubits and resonators. This creates the following terms in the interacting Hamiltonian,

$$\mathcal{H}_I = g(a^{\dagger}_{signal}a_{coupling} + a^{\dagger}_{drive}a_{coupling} + a^{\dagger}_{storage}a_{coupling} + a^{\dagger}_{cooling}a_{coupling}) + h.c. \quad (6)$$

This Hamiltonian allows for the following higher order terms to realise the desired interactions, as is shown below.

$$\mathcal{H}_{int1} = g^4(a^{\dagger}_{storage}a_{coupling})(a^{\dagger}_{cooling}a_{coupling})(a^{\dagger}_{coupling}a_{signal})(a^{\dagger}_{coupling}a_{drive}) \quad (7)$$

$$\mathcal{H}_{int3} = g^4(a^{\dagger}_{signal}a_{coupling})(a^{\dagger}_{signal}a_{coupling})(a^{\dagger}_{coupling}a_{storage})(a^{\dagger}_{coupling}a_{drive}) \quad (8)$$

Note that these interactions are suppressed by the frequency of the coupling resonator and therefore the frequency of the coupling resonator must be made small. It is also important to note that both of these interactions above are dependent on either an excitation in the signal or an excitation in the storage resonators, and therefore no interaction will occur if a signal was not present at the start. Other interactions allowed by this interaction Hamiltonian will be suppressed through tuning the qubit and resonator frequencies.

3 Design Problems

Energy conservation requires that the starting energy of an interaction is equal to the final energy. These requirements restrict the frequencies of the resonators such that,

$$\mathcal{H}_{int1} \implies \omega_{storage} + \omega_{cooling} = \omega_{signal} + \omega_{drive} \quad (9)$$

and for the second interaction,

$$\mathcal{H}_{int3} \implies \omega_{signal} + \omega_{signal} = \omega_{storage} + \omega_{drive} \quad (10)$$

However, combining these equations shows that these two constraints also imply the third constraint,

$$\mathcal{H}_{int1} + \mathcal{H}_{int3} \implies \omega_{signal} + \omega_{cooling} = 2\omega_{drive} \quad (11)$$

This third equation would allow for drive excitations to be converted into signal excitations even if no signal was present, allowing for leakage from the drive to signal resonators. This problem can be solved by preventing the drive resonator from being excited twice through restricting the drive resonator to a two-level system, or turning the drive resonator into a qubit.

Additionally, it was found during simulation of the amplifier in QuTiP that the second excited state of the coupling resonator allowed for destructive interference of the third interaction. This prevented a storage and drive excitation from being converted into two signal excitations. Similarly to the previous case, this was solved by restricting the coupling resonator to a two-level system.

Lastly, the cooling resonator only has to remove a single excitation per cycle of operation. Limiting the cooling resonator to a two-level system helps ensure that only a single excitation will be removed each round and improves the performance of the simulated circuit.

4 The Circuit

The necessary interactions can be realised in the following circuit,

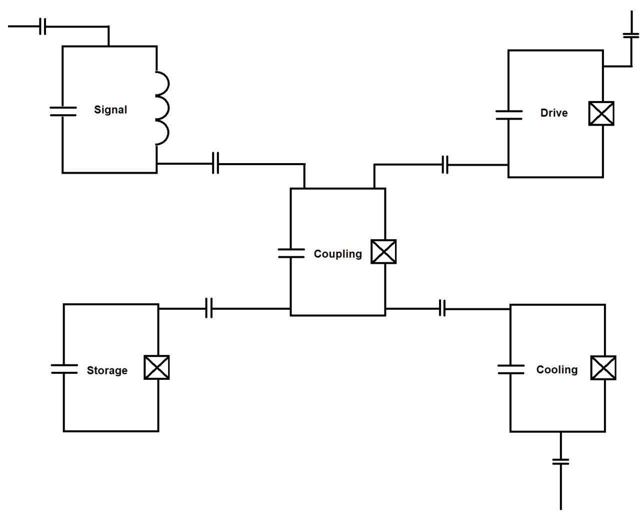

Figure 1. The amplification circuit. It contains a resonator for the signal, and four qubits corresponding to the drive qubit, coupling qubit, storage qubit, and cooling qubit.

This circuit has many tunable parameters. Among these parameters are the coupling strengths of each element to the coupling qubit, the frequencies of the qubits (subject to the constraints of the previous section), and the cooling strength. The results presented here were discovered by manually tuning these parameters. It may be possible to obtain an increase in the efficiency of amplification through a careful search of these parameters.

To understand the function of the circuit, we present the operational steps of the amplifier graphically.

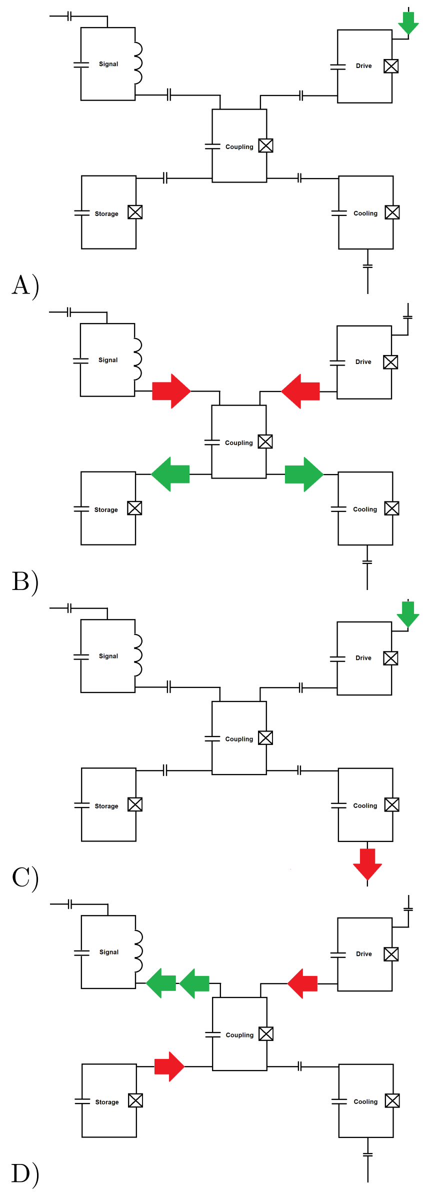

Figure 2. The steps of operation of the amplification circuit. A) the drive qubit is excited. B) A drive and signal excitation is converted to a storage and cooling excitation. C) The drive qubit is excited again while the cooling qubit loses an excitation. D) The storage excitation and drive excitation are converted into two signal excitations.

We can simulate each of these steps individually in QuTiP to examine their behavior. Here we display the two interactions previously shown.

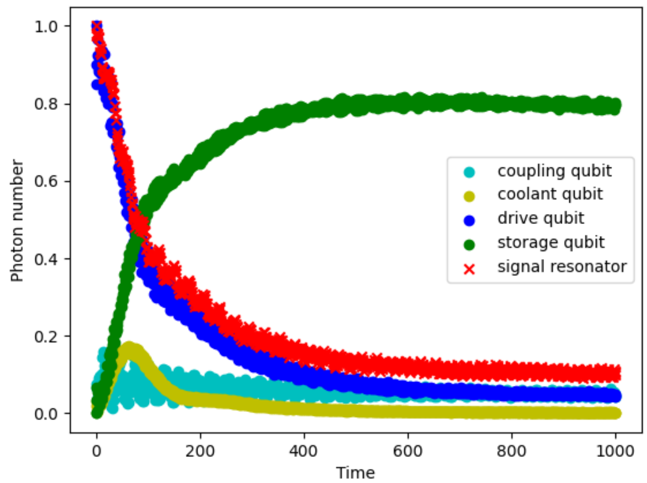

Figure 3. The first interaction of the quantum amplification circuit $\mathcal{H}_{int1}$. A drive excitation (blue) and signal excitation (red) are transformed into a storage excitation (green) and a cooling excitation (yellow) which is quickly removed.

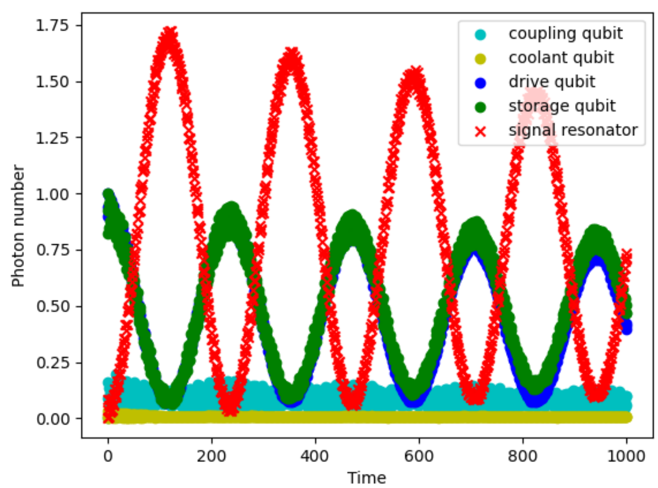

Figure 4. The second interaction of the quantum amplification circuit $\mathcal{H}_{int3}$. A drive excitation (blue) and a storage excitation (green) are turned into two signal excitations (red).

This should also be compared to the scenario where there is no signal excitation to begin with, showing that there is no leakage into the signal resonator.

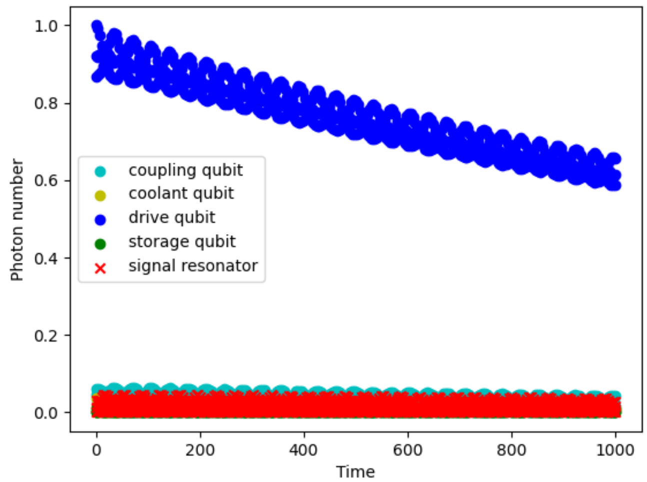

Figure 5. Step 2 of the quantum amplification circuit when no signal is present. It can be seen that the drive qubit does not significantly excite any other circuit elements.

Here are the results of the operation of the whole operation of the circuit as simulated in QuTiP.

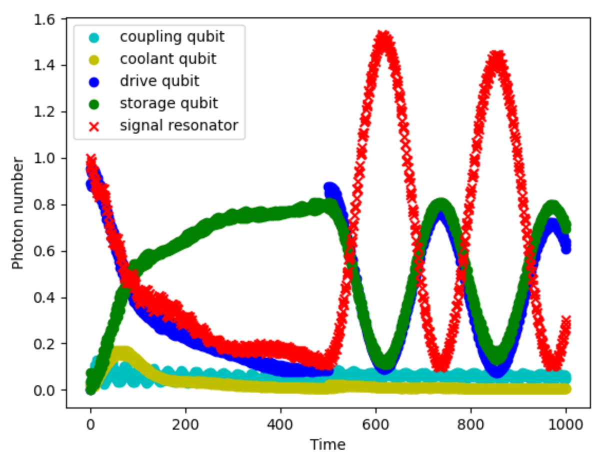

Figure 6. A full cycle of amplification from the quantum amplifier circuit. It can be seen that the first interaction is from timestamp 0 to 500 and the second interaction is from timestamps 500 to 600.

From this simulation, we can see that the first round of amplification is $\approx 80\%$ efficient. We can also investigate the case in which there is no signal present in the signal resonator to demonstrate that the signal resonator is not excited solely by the drive qubit.

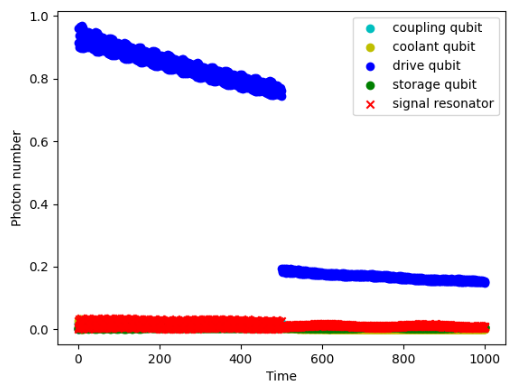

Figure 7. A full cycle of amplification from the quantum amplifier circuit where signal was not present. It can be seen that essentially no excitations leak into the signal resonator.

Next we would like to test the performance of this design across multiple rounds of amplification. However, the results of the simulation of a second round of amplification in QuTiP show that this does not increase the expected number of excitations in the signal resonator, as is shown in Figure 8. This is due to leakage from the signal or storage qubit out through the cooling qubit. Therefore, increasing the efficiency of the circuit will likely increase the expected number of photons in the amplification resonator.

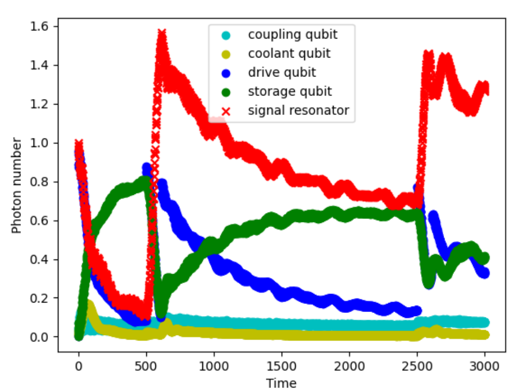

Figure 8. Two full cycles of the amplifier. The decrease in the average excitation number in the signal resonator is due to excitations leaking out of the cooling resonator.

5 Conclusion and Future Directions

In this paper, a circuit which is able to amplify quantum signals is designed and simulated in QuTiP. The design steps taken to engineer this quantum system were shown, starting from identifying the desired interaction, which was consequently broken down and engineered into multiple interactions. The circuit was additionally made to be irreversible through the introduction of a cooling qubit.

The circuit presented here suggests a framework for beginners to engineer quantum systems through follwing excitations in resonators and qubits rather than engineering a specific Hamiltonian with capacitors, inductors, and Josephson junctions. Engineering superconducting circuits starting from resonators and qubits arguably allows for a more intuitive description of the interaction and may allow for future quantum computer engineers to design circuits from more familiar circuit elements rather than requiring construction of a complex Hamiltonian from the fundamental circuit elements.

Further research could allow for multiple rounds of amplification by increasing the efficiency of the proposed circuit through tuning coupling strengths. Additionally, it may be found that, similarly to how JTWPAs needed phase matching to become more efficient [2], multiple rounds of amplification in the current design will need additional steps to prevent destructive interference in the amplifier circuit.

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